1. (4%) How many number of bits do you need to change in order to convert integer 31 to 14 and why?

2. (4%) Briefly explain the difference between the concepts of “overriding” and “overloading” in object-oriented programming?

3. In order for a deadlock to occur, you must have the following four conditions met. Deadlock prevention essentially entails removing one of the conditions. Briefly explain (c) and (d).
   (a) Mutual Exclusion: only one process can use a resource at a given time.
   (b) Hold and Wait: processes already holding a resource can request new ones.
   (c) (4%) No Preemption:
   (d) (4%) Circular wait:

4. The heap data structure can efficiently implement a priority queue that holds at most *n* elements at any point in time. Give the running time (in the Big-O notation) of the following operations and briefly explain why.
   (a) (2%) Insert(*H*,*v*) inserts the item *v* into heap *H*:
   (b) (2%) FindMin(*H*) identifies the minimum element in heap *H* but does not remove it:
   (c) (2%) Delete(*H*,*i*) deletes the element in heap position *i*:
   (d) (2%) ExtractMin(*H*) identifies and deletes the minimum element from heap *H*:

5. (6%) Write codes (for example, in C++) to complete the following task: if an element in an *m* x *n* matrix is 0, its entire row and column is set to 0.
   (4%) What is your running time (in the Big-O notation)?

6. In highly reliable computers, *residue codes* are sometimes used to enable checking of the basic arithmetic operations. The residue of a number *N* is defined by
   $R(N) = N \bmod p$, where *p* is called the *modulus*, that is the remainder after dividing a number *N* by another number *p*.
   (a) (5%) Construct the addition table for modulus 5 arithmetic using binary notation.
   (b) (5%) Construct the multiplication table for modulus 3 arithmetic using binary notation.

7. (10%) Convert the following decimal numbers to their pure binary equivalent:
   (a) 2397.55 (b) 0.79

8. (7%) The seed values F_1 =0, F_2 =1 and the recurrence relation is $F_n = F_{n-1} + F_{n-2}$, for $n > 2$.
   Please write a recursive function subprogram to compute the nth number F_n where n is assumed to be integer and nonnegative.

9. (6%) Prove the identities algebraically and then by perfect induction using a truth table.
   $B + \overline{A}C = (A + B + C)(\overline{A} + B + C)(\overline{A} + B + \overline{C})$

10. For every tree $T = (V, E)$, prove that (a) (5%) $|V| = |E| + 1$, and that (b) (5%) if $|V| \geq 2$, then T has at least two vertices with degree 1.

11. Some invoice (發票) administration system rewards invoice recipients according to the 8-digit invoice numbers (00000001 ~ 99999999). An 8-digit number $A[1:8]$ is randomly drawn. As shown in the following table, different categories of awards are given based upon the matching between invoice number $X[1:8]$ and target number $A[1:8]$. Write a function that accepts input A and X and reports the award for the number X. (8%)

1st place ($1,000,000)	$X[i]$=$A[i]$ for all i =1, 2, …, 8.
2nd place ($500,000)	$X[i]$=$A[i]$ for all i =2, 3, …, 8.
3rd place ($100,000)	$X[i]$=$A[i]$ for all i =3, 4, …, 8.
4th place ($50,000)	$X[i]$=$A[i]$ for all i = 4, 5, 6, 7, 8.
5th place ($10,000)	$X[i]$=$A[i]$ for all i =5, 6, 7, 8.
6th place ($5,000)	$X[i]$=$A[i]$ for all i =6, 7, 8.

12. (Stable Marriage Problem) Consider n men and n women for match-making of n pairs of opposite sexes. Each person has a preference list of all members of the opposite sex. The following instance (next page) consists of 5 men and 5 women. By his preference list, man 1 ranks woman 2 the highest, woman 3 the second, and so on; in woman 1's preference list, she prefers man 5 the highest and man 2 the least. The problem is to match n disjoint pairs of men and women. A matching is called *stable* whenever it is not a case where a man m of some pair prefers a woman w of another pair and the woman w also prefers the man m more than her current partner.

Men	Preference lists
1	2, 3, 4, 5, 1
2	1, 5, 2, 4, 3
3	5, 1, 3, 4, 2
4	3, 2, 5, 1, 4
5	4, 5, 2, 1, 3

Women	Preference lists
1	5, 4, 1, 3, 2
2	2, 5, 3, 1, 4
3	1, 3, 2, 4, 5
4	4, 2, 5, 3, 1
5	1, 5, 4, 2, 3

(a) Is the matching {(man, woman): (1, 1), (2, 2), (3, 3), (4, 4), (5, 5)} stable? Please give your reasoning. (3%)

(b) Use ALGORITHM STABLE-MARRIAGE given in the following box to derive a stable matching of the above 5-man, 5-woman instance. The algorithm is based on the strategy that men propose to women and women make decisions on acceptance or rejection. (5%)

(c) What is the running time, in the Big-O notation, of the algorithm? (4%)

(d) Suggest applications of the stable marriage problem. (3%)

```
ALGORITHM STABLE-MARRIAGE
set all men and all women free;
while there exists a free man m who has a woman w to propose to
{
    let w be the highest ranked woman to whom man m has not yet proposed
    if w is free, then collect the pair (m, w);
    else /* some pair (m', w) already exists
        if w prefers m more than m'
            collect the pair (m, w) and set m' free
}
```